Level splitting at macroscopic scale

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Unexpected analogies have been demonstrated recently between the dynamical properties of quantum objects and those of walkers : a classical self-propelled wave particle association moving on a fluid interface. As shown previously, two walkers can form orbiting bound states with discrete diameters. Here we investigate these bound states when the underlying bath is rotating. The binding diameter varies continuously with the imposed rotation velocity. It increases when the fluid and the orbiting motion are in co-rotation and decreases otherwise. Using the classical analogy between the forces due to rotation and those due to a magnetic field, it is possible to compare this effect to the Zeeman splitting of the energy levels of atomic orbits.

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The possibility of analogies between quantum waves and fluid surface waves was first shown to be relevant and useful by M. Berry *et al.* [1] when these authors gave an intuitive interpretation to the Aharonov-Bohm effect [2]. Until then, this quantum effect had been considered strange: electrons traveling outside a solenoid containing a magnetic field are scattered even though they travel in magnetic-field-free regions of space. Berry *et al.* showed that this effect is linked with the formation of a dislocation in the front of the Schrödinger wave. These authors then used the formal similarity that exists in classical physics between the effect of rotation on moving masses and the effect of a magnetic field on moving charges. With this analogy they showed that the same type of dislocation is created in the fronts of surface waves when they meet a vortex. In this system, the equivalent of the vector potential is the fluid velocity and the formation of the dislocation is simply ascribed to the different advection of the wave front on both sides of the vortex. This effect has now been thoroughly investigated for surface waves [3, 4] as well as for acoustic waves [5].

An even larger range of possible analogies between quantum physics and fluid mechanics was opened when a hydrodynamic system presenting wave-particle duality at macroscopic scale was introduced [6, 7]. On the surface of a vertically vibrated liquid bath, a bouncing droplet can couple to the surface waves it emits and become a propagative *walker*. Using this system, quantum-like single particle diffraction and interferences were observed [8] as well as a tunneling effect [9]. Furthermore, using the analogy between magnetic field and rotation it was found that quantized orbits could be obtained if the fluid bath was set in rotation. These orbits were shown to be the classical analogs of the quantum Landau orbits in a magnetic field [10].

Another previous result is that, in the absence of rotation, two identical walkers can bind to each other into orbiting states [7] (see fig. 1 (a-b)). In these bound states, each droplet is submitted to a supplementary force due to the wave-mediated interaction and thus trapped in the wavefield created by its partner. We can recall that both droplets bounce at the Faraday frequency f_F, which is half the forcing frequency $f_0 = 2f_F$. For this reason each droplet can have two possible phases and a pair of droplets can have either a synchronous bouncing or have opposite phases; two families of bound states are thus observed. The orbit diameters d_n are discrete and scaled on the Faraday wavelength λ_F and given by [7]:

$$d_n = (n - \epsilon_0)\ \lambda_F \tag{1}$$

where $\epsilon_0 = 0.21$ and n is an integer if the droplets are in phase ($n = 1, 2...$), a half-integer if they are of opposite phases ($n = \frac{1}{2}, \frac{3}{2}...$). We also measured V_n the walkers velocity orbiting on the n-th bound state. It is minimal for $n = 1/2$ and slowly increases with n reaching asymptotically the free velocity of the walker. The minimal speed of the walker $V_{1/2}$ is about 20% smaller than V_{free}. In the present work, we study the effect of an externally imposed rotation on the orbits. Then, using the analogy between magnetic field and rotation, we compare our results to the effect of a magnetic field on atomic orbits.

Experiments – The experimental set up is identical to that used in [10]. We use a silicon oil with viscosity $\mu = 20.10^{-3}$ Pa.s, density $\rho = 965$ kg.m^{-3} and surface tension $\sigma = 20.9$ mN.m^{-1}, subjected to a sinusoidal vertical acceleration $\gamma = \gamma_m \sin 2\pi f_0 t$. The forcing frequency is $f_0 = 80$ Hz. The droplets have a diameter $D_G = 0.8$ mm, a mass m_G and bounce with the Faraday frequency $f_F = f_0/2$. The measured Faraday wavelength $\lambda_F = 4.75$ mm is in good agreement with prediction from the surface wave dispersion relation for a deep bath. As shown on Fig. 1(c), the cell can be set into counterclockwise rotation at an angular velocity Ω, ranging from 0 to 5 Hz.

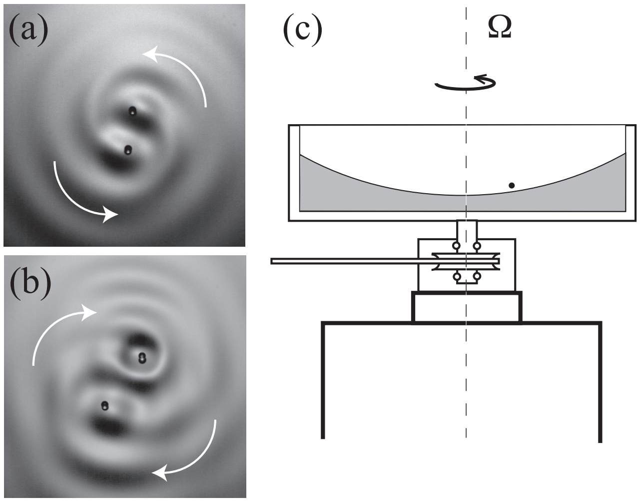

FIG. 1: (a) Snapshot of two counterclockwise orbiting walkers with $n^+ = (1)^+$. (b) Snapshot of two walkers orbiting clockwise with $n^- = (1.5)^-$. (c) Experimental set-up in rotation. The angular velocity Ω can be tuned with a motor.

We create two identical droplets and check that in isolation they walk at the same free velocity. We then organize collisions by which they bind to each other. We identify the order of the resulting orbiting state and study the effect of rotation. This being done, we disturb the system with a needle so as to separate the droplets and bind them again in a different state. Note that there are two possible states for each value of n: the co-rotating state that we will denote n^+ and the counter-rotating one that we will denote n^-. In our experiments, the bath is rotating counterclockwise, thus n^+ orbits are rotating counterclockwise, n^- clockwise. Two examples of these orbits are shown. Fig.1(a) is a $n^+ = (1)^+$ orbit and Fig.1(b) is $n^- = (1.5)^-$. Using this procedure we were able to investigate the effect of rotation on all the bound states of the same pair of droplets.

The bath being set into rotation, the trajectories of the two drops are recorded in the laboratory frame of reference. Both are epicycles [fig. 2 (a)]. From these data, the trajectories in the rotating frame can be reconstructed and they are found to be circular. The vector $\vec{d}$ joining the two walkers has a constant modulus $d_n(\Omega)$ and rotates with a constant angular velocity ω_{orb} [see fig.2 (b)]. Two cases must be distinguished, depending of the sign of the orbital motion. When the orbit and the bath are contra-rotative (n^+) the motion of the two droplets is slowed in the laboratory frame, the orbit is stabilized and its diameter decreases. In the co-rotative case (n^-), the increase of Ω results in both the orbital velocity and orbit diameter. When a limit value of Ω is reached, the orbital state becomes unstable and breaks down.

We repeated this experiment using all the possible orbits of order n of the two droplets. The evolution of the diameters d_n with respect to Ω are shown on figure 2 (c).

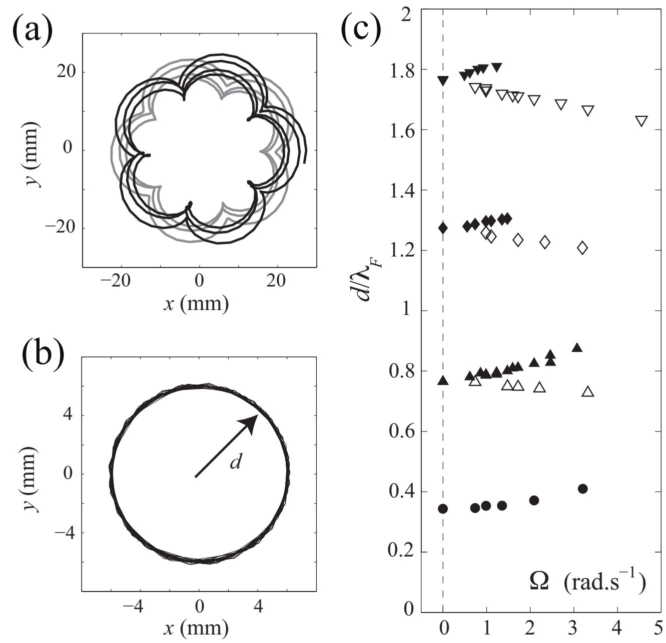

FIG. 2: (a) Recorded trajectory in the laboratory frame of reference for two walkers on a $n^- = (1.5)^-$ level and $\Omega = 0.99$ rad.s^{-1}. (b) Time evolution of the vector $\vec{d}$ joining the two walkers showing the diameter of the orbit. $\vec{d}$ has a constant modulus and rotates with a constant angular velocity $\omega_{orb} = 5.44$ rad.s^{-1}.(c) The evolution of the diameters d_n with respect to Ω for different values of n. The open symbols correspond to contra-rotation (n^-), the black ones to co-rotation (n^+).

We observe experimentally that all the d_n vary linearly with Ω: they decrease for all the n^- and increase for all the n^+ states. The observed slope increases with n. In the following we write that $d_n(\Omega) = d_n(0) + \delta_n(\Omega)$ where δ_n is the shift relative to the diameters observed in the absence of rotation.

Probing the binding force – We use the external field Ω as a control parameter to probe the interaction force between two walkers. Without rotation, the diameters d_n are given by a balance between the centrifugal force $F_c = 2m_G {V_n}^2/d_n$ and the wave mediated interaction force $F_{\rm int}$. When the bath is set in rotation, the Coriolis force $F_{\rm Cor} = 2m_G\Omega V_n$ orthogonal to motion modifies this equilibrium. The shift δ_n can be related to a new equilibrium between $F_{\rm int}$, $F_{\rm c}$ and $F_{\rm Cor}$. As the rotation doesn't induce any change of V_n, δ_n induces a typical variation of the centrifugal force $\delta F_c < 8.10^{-8}$ N which is much smaller than the applied Coriolis force $F_{\rm Cor} \simeq 2.10^{-7}$ N. Thus, we can neglect the variation of F_c and we obtain a simple expression for $F_{\rm int}$:

$$F_{\rm int}(d_n + \delta_n) = 2m_G(\Omega V_n + \frac{{V_n}^2}{d_n}) \qquad (2)$$

The deduced value of $F_{\rm int}$ from the experimental data is shown in Fig. 3 (a)). We observe that $F_{\rm int}$ oscillates with

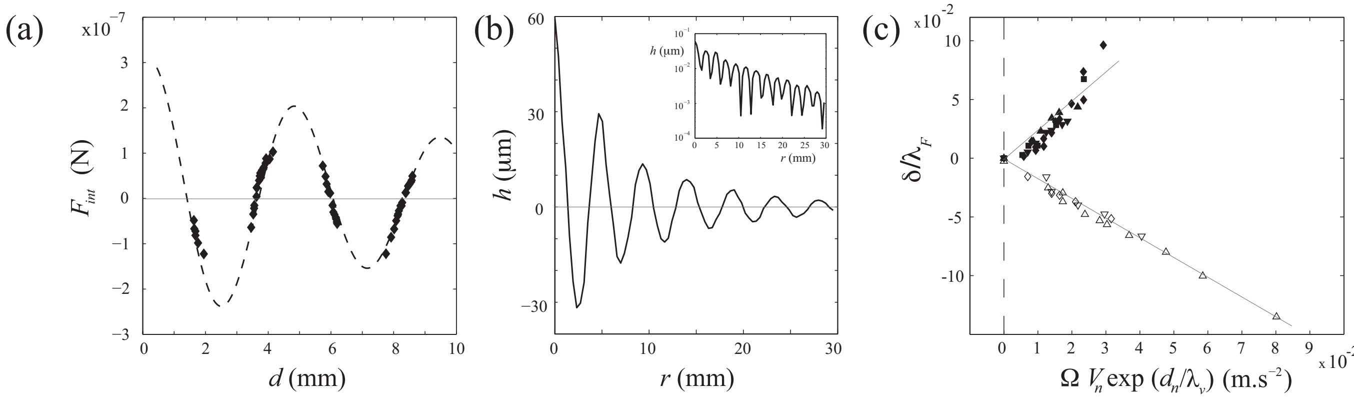

FIG. 3: (a) Measured interaction force F_{int} as a function of d. The dashed line is guide for the eye. (b) Experimental measurements of the wave created by a walker in the direction orthogonal to motion. The inset in semi-logarithmic scale shows an exponential decay of the wave. The characteristic length of decay is $l_v \simeq 2\lambda_F$. (c) Experimental data δ_n as a function of Ω with the scaling given by equation 5. All data collapse on the same master curve.

the same wavelength as the surface waves.

The interaction between the two walkers is related to the shape of the wavefield they generate [7, 11]. Using a Synthetic Schlieren technique [12], the surface height $h(d)$ has been measured in the direction orthogonal to motion for a free walker (see fig. 3 (b)). It has a spatial pseudo-period λ_F and decays exponentially because of viscous damping with a characteristic damping length $l_v \simeq 2\lambda_F$. The interaction force is given by the slope of the surface at the bouncing point [7]:

$$F_{\rm int}(d) = m_G \gamma_m \frac{\tau}{\tau_F} \vec{\nabla} h(d) \sim m_G \gamma_m \frac{\tau}{\tau_F} \frac{A_w}{\lambda_F} \tag{3}$$

with τ/τ_F the ratio between contact time τ during bouncing and Faraday period τ_F and A_w the amplitude of the surface wave. Typical values are $\gamma_m = 5$ g, $\tau/\tau_F \sim 0.2$ and $A_w/\lambda_F \sim 0.01$. This leads to $F_{\rm int} \sim 2.10^{-7}$ N, in good agreement with the measured amplitude of the force. We thus assume that one can write :

$$F_{\rm int} = F_0 \ \exp\left(-d/l_v\right) \sin\left(\frac{2\pi d}{\lambda_F} + \varphi\right) \tag{4}$$

Such an expression for $F_{\rm int}$ allows for the numerical simulations of discrete orbits of walkers [7], but also of quantized self-orbits [10]. From this description we can deduce a theorical scaling for the shift δ with a linear dependency in Ω. Expanding eq. 2 with eq. 4 we get :

$$\frac{\delta}{\lambda_F} = \frac{m_G}{\pi F_0} \ \Omega \ V_n \exp\left(\frac{d_n}{l_v}\right) \tag{5}$$

Figure 3 (c) shows the experimental data rescaled using this expression. This scaling provides a good collapse of the diameter shift for each n bound state. The predicted slope $m_G/(\pi F_0) \simeq 3$ m^{-1}.s^2 is in qualitative agreement with the experimental one (see fig. 3(c)), confirming the validity of the previous analysis. The observed scaling is linked with the spatial decay of the waves due to viscosity. Note that in a non-dissipative system, the spatial decay would be given by Bessel functions, changing the final scaling.

Analogy with Zeeman splitting – The relation $2\ \mathbf{\Omega} = \nabla \times \mathbf{U}$ between the fluid velocity $\mathbf{U}$ and the vorticity $\mathbf{\Omega}$ corresponds to $\mathbf{B} = \nabla \times \mathbf{A}$ in electromagnetism [1]. The magnetic field $\mathbf{B}$ is thus formally equivalent to the vorticity $2\mathbf{\Omega}$ and the vector potential $\mathbf{A}$ corresponding to the velocity field $\mathbf{U}$ of the waves. This equivalence has previously been used to give a complete interpretation to the quantization of circular orbits for a single walker on a rotating bath [10], reminiscent of the Landau levels at quantum scale [13].

Our experiments show that, for a system of two droplets with a angular momentum $L_n^{\pm} = \pm m_G d_n V_n$, an external rotation Ω induces a linear shift of the orbit diameter $d_n(\Omega)$. Using the correspondence of the equations between fluid mechanics and electromagnetism, we seek a similar situation in the quantum world. Our system turns out to be equivalent to a rotating electrical charge e subjected to a constant magnetic field $\mathbf{B}$. In that case, it is possible to define a magnetic interaction energy :

$$E_{\rm mag} = \frac{e}{2m_e}\mathbf{L}.\mathbf{B} \tag{6}$$

where m_e denotes the mass of the particle and L its angular momentum. If we consider an electron with an energy level E with a characteristic frequency $\omega_E = E/\hbar$ the magnetic interaction induce a shift frequency $\delta\omega_E$ given by :

$$\delta\omega_E = \frac{e}{2m_e}\frac{\mathbf{L}\cdot\mathbf{B}}{\hbar} \tag{7}$$

This effect is responsible for Zeeman splitting in quantum mechanics [14]: two states with different values of $\mathbf{L}$ will have different characteristic frequency when $\mathbf{B} \neq 0$. In

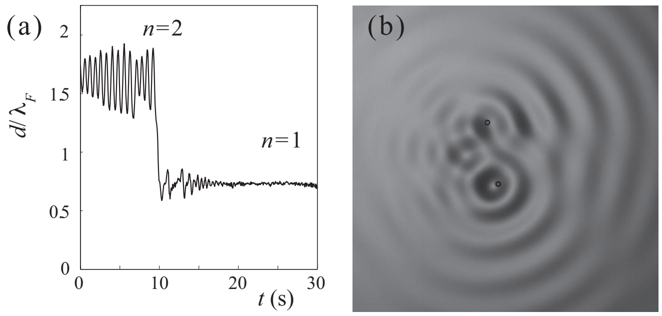

FIG. 4: (a) Evolution of the adimensionnal distance d/λ_F between two walkers for a rotation rate $\Omega = 4.3$ rad.s^{-1}. After transient oscillations, we observe an abrupt transition from $n = 2$ to $n = 1$. (b) Snapshot of the complex structure of the unsteady wavefield during the transition.

our classical system, we can also define a potential energy $E_{\mathrm{Coriolis}} = 2\mathbf{L}_n \cdot \mathbf{\Omega}$ for a walker submitted to a vorticity Ω. As the potential energy E_{Coriolis} is formally equivalent to E_{mag}, our experimental results directly compare to Zeeman splitting. Each orbital level of the discrete set $n = 1/2, 1, 3/2, 2, ...$ can have two symmetrical states $L_n^{\pm}$. When no external field is applied, these two states cannot be physically distinguished. When $\Omega \neq 0$, the system is plunged in a potential $2\mathbf{L}_n \cdot \mathbf{\Omega}$ and we observe a linear shift δ_n, with the sign of this shift determined by the sign of the energy variation. We can compute the rotation frequency $\omega_n(\Omega) = V_n/d_n(\Omega)$ and develop this expression at first order to get the frequency shift $\delta\omega_n^{\pm}$:

$$\delta\omega_n^{\pm} = \frac{V_n \delta_n(\Omega)}{d_n(0)^2} \tag{8}$$

From eq. 5, $\delta\omega_n^{\pm}$ can be written as a linear function of the external field Ω :

$$\delta\omega_n^{\pm} = \pm\alpha_n\Omega \tag{9}$$

where α_n depends only on the order n of the binding level. We thus have an analogy with Zeeman splitting. We note that in both cases, the shift depends on the n order of the level, but in a more complex way in our experiment. We believe that this is due to the fact that, in our system, the distances are quantized, whereas in quantum mechanics, this is true for energy and frequency.

Inter-level transitions – Can we use the external potential E_{Coriolis} to modify the bound state? We prepare a $n^- = (2)^-$ orbit, and submit it to an intense counter rotation. The distance between the two droplets first decreases according to eq. 5 and then presents some oscillations [fig. 4(a)]. For a large enough value of Ω, these oscillations are amplified and the level $n = 2$ becomes unstable: the distance between the two walkers decreases abruptly, and they set on the $n = 1$ level. During the transition, the wavefield around the two walkers present a strong asymetry (see fig. 4(b)). Note that such transitions are submitted to selection rules. The direction of rotation is preserved, so that transitions from levels n^- to n^+ are forbidden. Similarly, the relative phase of the two bouncing droplets is also preserved so that transitions are forbidden between integer and semi-integer orders: a drop cannot transit from $n = 2$ to $n = 3/2$. When the two droplets are bouncing in phase, the $n = 1$ level is the fundamental level, while it is $n = 1/2$ for opposite phase droplets. The oscillations observed prior to transition could be analogous to Rabi oscillations in between two resonant levels. Both the oscillations observed before the transition and the structure of the waves emitted during the transitions should be studied in order to understand the very nature of these events. We reserve these studies for a further work.

Conclusion – We have probed the response of orbital bound states of walkers when subject to a Coriolis force. This gave us a direct measurement of the interaction force between two walkers. The external potential applied on the bound states induces a level splitting at macroscopic scale: two symmetrical set of levels n^+ and n^- can be distinguished when $\Omega \neq 0$. Through analogy connections, electromagnetism and hydrodynamics, we have shown that this experiment is reminiscent of Zeeman splitting in quantum mechanics. As the external potential allows for shifts in the energy of the system, we can induce transitions between levels at the macroscopic scale. The mechanism of these transitions should be studied in detail.

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