Probabilities and trajectories in a classical wave-particle duality

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Abstract. Several recent experiments were devoted to walkers, structures that associate a droplet bouncing on a vibrated liquid with the surface waves it excites. They reveal that a form of wave-particle duality exists in this classical system with the emergence of quantum-like behaviours. Here we revisit the single particle diffraction experiment and show the coexistence of two waves. The measured probability distributions are ruled by the diffraction of a quantum-like probability wave. But the observation of a single walker reveals that the droplet is driven by a pilot wave of different spatial structure that determines its trajectory in real space. The existence of two waves of these types had been proposed by de Broglie in his "double solution" model of quantum mechanics. A difference with the latter is that the pilot-wave is, in our experiment, endowed with a "path memory". When intrusive measurements are performed, this memory effect induces transient chaotic individual trajectories that generate the resulting statistical behaviour.

1. Introduction

At the origin of this work is the discovery that it is possible, at classical scale, to obtain a steady regime of propagation for a structure associating a localized particle and a wave. The particle is a droplet bouncing on the vibrated surface of a liquid and the wave is the surface wave generated by the bouncing. It is known that when a liquid bath is vertically vibrated at a frequency f_0 with an acceleration amplitude larger than an onset value, the whole surface becomes unstable and a wave pattern of frequency $f_0/2$ forms spontaneously due to parametric forcing. This is the Faraday instability [1]. The dynamical coupling of the droplet to the wave occurs immediately below this threshold. For such a forcing and when the droplet has the correct size, its bouncing becomes sub-harmonic so that the droplet acts as an efficient local exciter of Faraday waves. In this regime the bouncing droplet and the wave it generates become phase-locked and the whole object, the drop and its associated wave, becomes spontaneously propagative [2,3]. In spite of the system being dissipative, this self-propagative structure is sustained. The bath oscillation provides energy both to the drop to maintain its bouncing and to the waves through

their parametric forcing.

We called these structures "walkers" and various experiments were undertaken to understand how the particle and the wave could have a common motion on the 2D surface of the bath. They revealed that some behaviours thought to be strictly reserved to quantum duality showed up at macroscopic scale in the dynamics of these objects. Single particle diffraction and interference were observed [4] as well as a form of tunneling effect [5]. Furthermore, when an applied transverse force gives a walker a circular motion, it was found that its possible orbits were quantized and an analogy existed with the quantum Landau orbits in a magnetic field [6]. All these results were shown to emerge from the spatio-temporal non-locality of the walker due to what we called its "wave-mediated path-memory" which is described in detail in references [6, 7]. At each collision with the interface the droplet emits a circular travelling capillary wave that triggers the formation of a localized mode of Faraday standing waves of wavelength λ_F. Because of the proximity of the instability threshold, these waves are sustained during "a memory time" τ that becomes large near the Faraday instability threshold [7]. As a result the wave field is the linear superposition of the successive Faraday waves emitted by past bounces. Its complex interference structure thus contains a memory of the recent trajectory. Furthermore, since the travelling waves move faster than the drop, the wave field also contains information about the obstacles that lie ahead. Hence, two non-local effects exist in the wave-field driving the motion of the droplet: the past bounces influence directly the present (direct propulsion) and the trajectory is perturbed by scattered waves from distant obstacles in a kind of echo-location effect. This interplay between the droplet motion and its associated wave field makes it a macroscopic implementation of a pilot-wave dynamics.

Such a system is reminiscent of the early de Broglie models for quantum systems [8-11]. In the present article we will limit ourselves to revisiting our diffraction experiment [4] in the terms used by de Broglie in his pilot-wave and double solution model [10, 11]. This will lead us to a discussion of the disordered trajectories generated by intrusive measurements and the resulting probabilistic behaviours.

2. The experiment and its results

In the experiment reported in ref. [4] we had investigated the diffraction of walkers as they pass through one slit. The reader can refer to this letter for the experimental set-up as well as for the precise description of the results. Here we will revisit these results by thought experiments in which the observational means are restricted. We can thus artificially introduce the measurement problem central to quantum physics. With this aim we first present the statistical results that can be obtained with reduced means.

2.1. The blind statistical results

The thought experiment is sketched in Fig. 1. We assume that we have a source of walkers and that the only information we can get about a walker is the position of the droplet *that we can only obtain by destroying it*. A single droplet trajectory cannot be tracked and the surface waves are not visible. We thus have to resort to statistical measurements. Between a source of walkers and a detector screen, we place a wall opened with a single slit of width L. The source is located sufficiently far so that the walkers it emits have a normal incidence ($\alpha_i = \pi/2$) on the wall. Their flux is homogeneous and they impinge with the same probability in all regions of the wall and the slit. The source is weak so that successive walkers cross the slit one at a time. On the screen placed at a distance behind the slit the impacts of the successive droplets accumulate. The experiment shows that a diffraction pattern builds-up. The resulting histogram (Fig.1) of droplet deviations α_d is well fitted by the modulus of the amplitude of diffraction of a plane
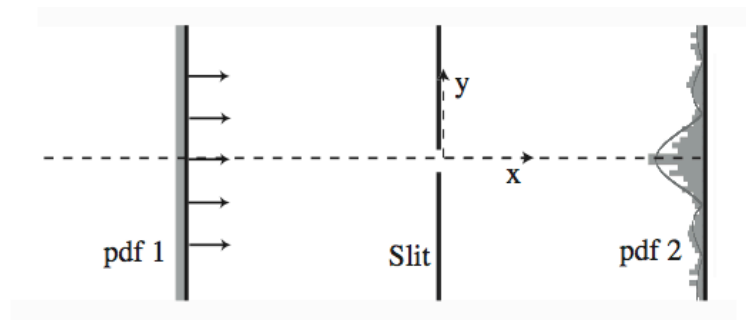
Figure 1. Sketch of the diffraction experiment with reduced means of observation. If only a detection of the locus of impact of the walkers on a screen is possible, the only experimental result is that the probability distribution function for the deviations is given by the diffraction of a plane wave of wavelength λ_F passing through the slit of width L.

wave of wavelength λ_F passing through the aperture of width L:

$$f(\alpha_d) = \frac{A\sin(\pi L\sin\alpha_d/\lambda_F)}{\pi L\sin\alpha/\lambda_F} \tag{1}$$

This means that the probability distribution of the deviations of a droplet is given by the diffraction of a plane wave [4]. This result is similar to what would be obtained with electrons or photons except that the distribution would then be given by the square of the wave amplitude.

2.2. Diffraction as an intrusive measurement
Passing through the slit can be considered as a means by which we measure a droplet's position in the y direction. Before crossing the slit, due to the homogeneity of the flow in the semi-infinite medium, a given droplet has an equal probability to be anywhere in the y direction and its velocity component V_y is zero. After having passed through the slit we know its position: it is located at the abscissa $y = 0$ with a precision $\delta y \sim \pm L/2$. This information has been obtained at the cost of a loss of definition on the transverse component V_y of its velocity. An uncertainty δV_y has appeared. As in quantum mechanics, this uncertainty has been generated by the intrusion of the measurement itself. The measured probability distribution functions, both before the slit and after it, are similar to those of a plane wave of wavelength λ_F. This wave is a probability wave: it plays a similar role to that of the wave function in quantum mechanics.

2.3. The non-intrusive observation
The elementary structures of our experiment are bouncing droplets and their associated surface waves. Measurements involving exclusively the interactions of this type of objects would certainly be intrusive. However in this system non-intrusive observations can be obtained with visual observation. By switching on the light, we can observe with the naked eye either the droplet only, or both the droplet and its wave field. The droplet alone appears as a point-like particle. Fig. 2 shows its recorded trajectories during two successive crossings of the slit. The trajectories are observed to be complex in the slit region. In spite of rather similar initial conditions they deviate differently.

The origin of such individual trajectories is revealed only when we observe both the particle and the fluid interface. As deduced from the blind experiments, the association of the drop with a wave of wavelength λ_F is responsible for the diffraction. However, the observed wave field is very different from the wave that determines the probabilities. No common plane wave is
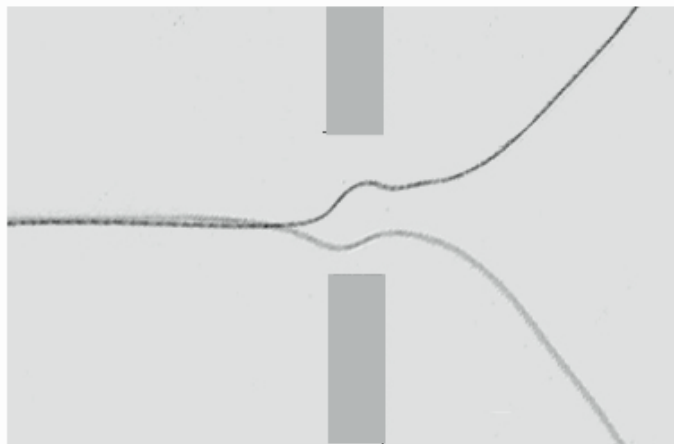
Figure 2. Two measured trajectories of single droplets passing through the slit. The similar initial conditions generate very different trajectories.

observed before the slit crossing and no common diffracted wave is observed behind it. Instead, the droplet is seen to be at the centre of a wave packet with a complex and constantly evolving structure (Fig. 3). The nature of this wave packet is now well-understood and described in Fort *et al.*[6].

The topography of the liquid surface is determined by the superposition of circular standing waves centred on the positions of the past bounces of the droplet. The global wave-field thus has an interference structure that contains a memory of the particle's recent trajectory. This is what we have called [6,7] the "wave-mediated path-memory" of the walker. The horizontal motion of the droplet is driven iteratively, bounce after bounce, by its coupling to this wave. At each collision with the interface the drop undergoes a damping due to viscous friction but it is simultaneously given a momentum increment by its shock with the slanted oscillating surface. The direction of the kick results from the local slope of the interface at the point of impact (see Fig. 4). The resulting velocity of the drop is thus :

$$m\frac{d\mathbf{r_i}}{dt} \propto \nabla\zeta(\mathbf{r_i}, \mathbf{t_i}) \tag{2}$$

where m is the mass of the droplet, $\mathbf{r_i}$ its position in the plane at the collision of order i and $\zeta(\mathbf{r_i}, \mathbf{t_i})$ the height of the liquid interface in $\mathbf{r_i}$ at time t_i.

This path memory model retains all the dynamical behaviour of the experimental walker. Its numerical implementation is described in detail in ref. [6,7]. It accounts for the spontaneous motion of the droplet as well as for the structure of the resulting pilot-wave field. It was used in ref. [4] to investigate the diffraction of walkers. Figure 5 shows the computed trajectories of individual walkers impinging on the slit and coupled to waves that have a large path memory. The individual trajectories (Fig. 5) confirm the main characteristics observed in the experiment.

- The droplets are deviated long before they reach the slit. Since the propagative front of the wave field moves faster than the droplet, it has an echo-location function. The standing Faraday waves that it generates contain information on the shape of the boundaries.

- The complexity of the trajectories in the region of the slit leads to their divergence. Droplets having very similar initial trajectories far from the slit can undergo different deviations. This
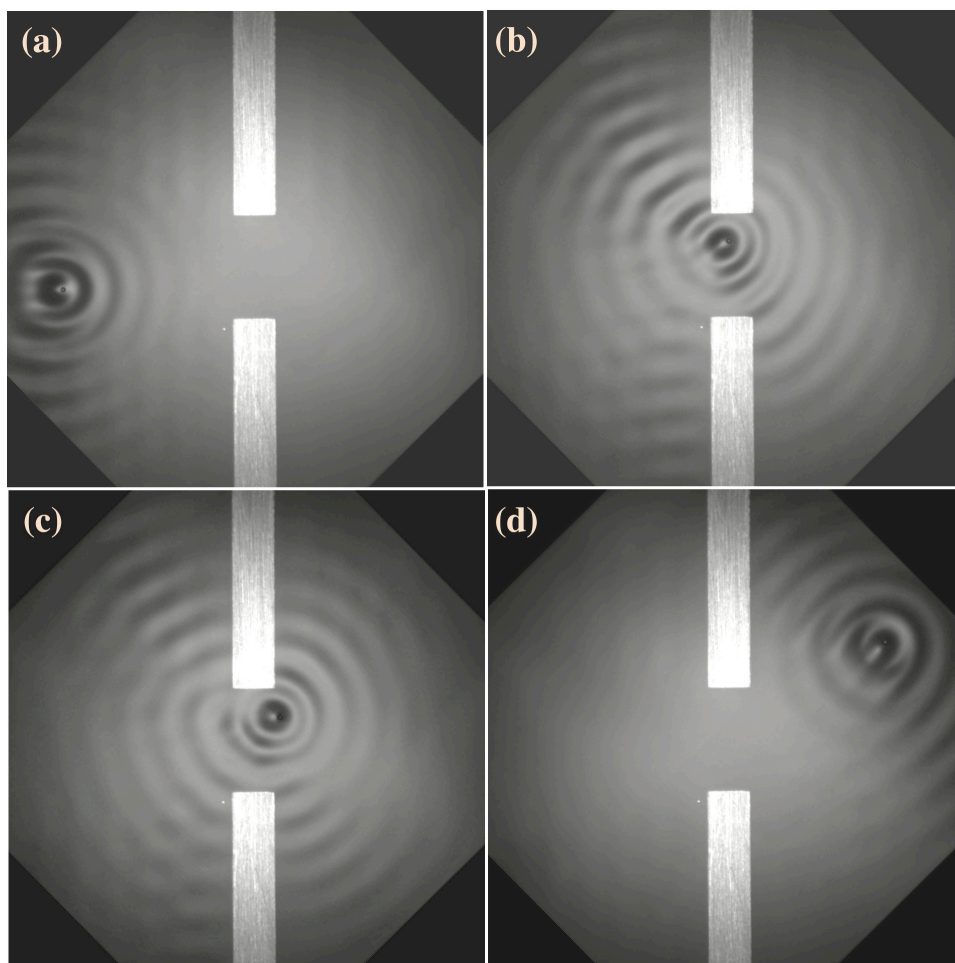
Figure 3. Four photographs of a walker as it passes through the slit. Note the complexity of the pilot wave field when the drop is in the slit region (b and c).
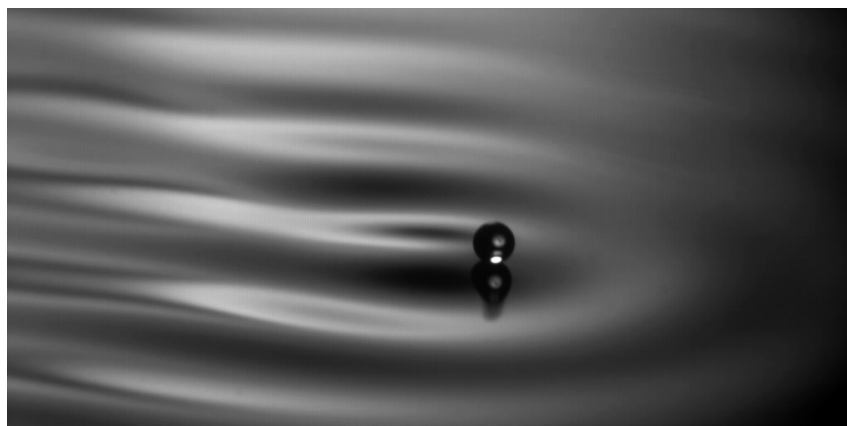

Figure 4. Close-up photograph of the collision of the droplet with the locally slanted interface due to the wave.
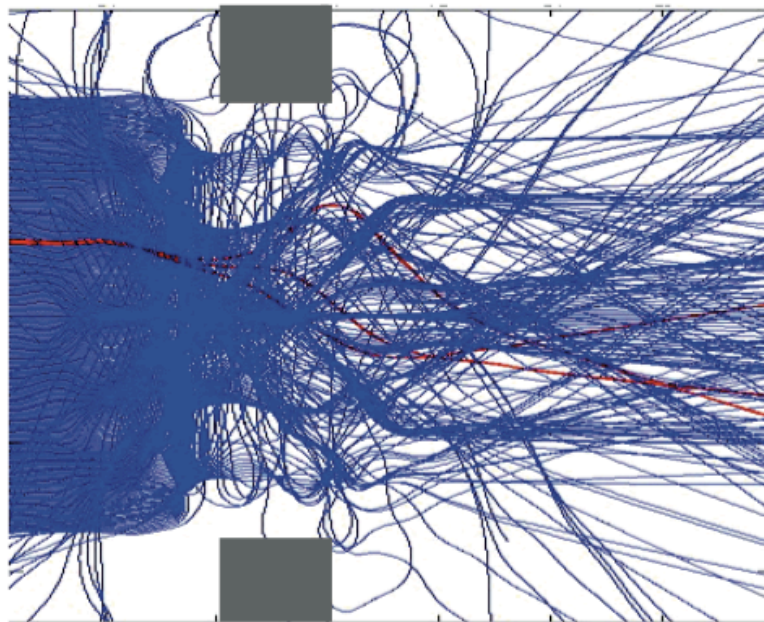
Figure 5. The simulation of the motion of a hundred walkers impinging on a slit. Note that, as in the experiment, some trajectories cross the symmetry axis of the apparatus.

appears as the source of the observed uncertainty.

- Some of the trajectories cross the symmetry line. A drop impinging on the left of the slit can be deviated to the right.

- Finally, in the numerical simulation, as in the experiment, a diffraction pattern of a plane wave emerges from the statistics of the deviations of a large number of independent realizations [4].

3. The relation with de Broglie pilot wave models
Our experiment is characterized by the coexistence of two waves. One is an individual physical wave excited by the drop that has a pilot-wave role during the trajectory in space and time. The second is not visible at any particular time but determines the probabilities of the measurements. This appears very close to the hypothesis of a double solution put forward by de Broglie with the aim of restoring determinism in quantum mechanics.

It is useful here to summarize very briefly the historical evolution of the pilot wave models of quantum mechanics usually gathered as "de Broglie-Bohm model". This latter terminology turns out to be misleading since the theoretical approaches of these two authors, though inter-related, are very different from one another.

3.1. de Broglie's pilot wave model
In his initial work in 1925, de Broglie [8] proposed that a particle of rest mass m_0 (such as an electron) has an oscillation at a frequency $\nu_0 = m_0 c^2 / h$. This singularity is surrounded by a standing wave-field of wavelength $\lambda_{dB} = h/mV$ formed by the superposition of an emitted wave radiated away (retarded wave) and a wave converging towards the particle (advanced wave). In

his model the particle is guided by the wave, its velocity being linked to the wave by

$$m\frac{d\mathbf{x_i}}{\mathbf{dt}} \propto \nabla_\mathbf{i}\mathbf{S} \qquad (3)$$

where S is the phase of the guiding wave.

Such an association of the particle to a wave field was meant to introduce the possibility of non-newtonian behaviours. This was summarized by de Broglie in the following terms [10]: *"the particle's motion . . . guided by the wave. . . would not follow the laws of classical mechanics according to which the particle is subject only to the action of forces which act on it along its trajectory, and does not suffer any repercussion from the existence of obstacles which may be situated far away outside its trajectory. In my (. . .) conception, on the contrary, the movement of the singularity should experience the influence of all the obstacles which hinder the propagation of the wave with which it is connected. This circumstance would explain the existence of the phenomena of interference and diffraction".*

Inspired by this work, Schrödinger initially introduced his equation while seeking de Broglie's wave. It was only later that Born demonstrated that the Schrödinger equation was related to the probability of presence. de Broglie then suggested in 1927 [9] that in all quantum systems two wave fields have to be considered:

- The first wave-field (that he called **u**) is generated by the particle. It is a real wave that has a singularity at its core. (In later versions he proposed that the core region could be strongly non-linear). For de Broglie this wave-particle structure had well-defined trajectories in the physical space. The particle could be submitted to forces but its association to the wave also gave it a non-local sensitivity to the environment and a non-newtonian behaviour.

- The second wave-field is ψ the linear and smooth solution of the Schrödinger equation that is related to the statistical density of probability of presence.

3.2. Bohm's pilot wave model

Much later, in 1952, David Bohm [12] revisited the idea of a pilot wave and investigated the situation where the wave solution of the Schrödinger equation would be the pilot wave. This idea had been briefly considered by de Broglie himself but then renounced [13]. Bohm used the Madelung transformation of the Schrödinger equation to obtain the following equation of motion:

$$m\frac{d\mathbf{x_i}}{dt} \propto \nabla_\mathbf{i}\mathbf{S}_\psi \qquad (4)$$

where is the phase of the Schrödinger wave function. This equation is similar to eq.3, but deceptively so, since these two equations are related to two different wave fields (**u** and ψ, respectively). In the Bohmian model where the wave function is decoupled from the trajectory itself, a slightly modified Newtonian mechanics can be recovered by addition of a "quantum potential".

The Bohmian trajectories, being computed from Schrödinger's equation, remain the trajectories of probability densities, rather than those of individual particles. This is the reason for which de Broglie wrote shortly after Bohm's publication [10] : *". . . David Bohm took up the pilot-wave theory again. His work is very interesting in many ways (. . .) But since Bohm's theory regards the wave ψ as a physical reality, it seems to me to be unacceptable in its present form".* This led de Broglie to restate [11] his "double solution" hypothesis where a wave field **u** is associated to a given particle and guides its motion. However, little progress was done along this line. The **u** waves remained undetected and it was unclear how they could generate the random behaviours responsible for the statistics. For these reasons, research in this domain appeared to have stalled. By contrast, the recent developments of the Bohmian approach have permitted the computation of trajectories in such situations like the interference and diffraction of particle beams [14]. In

a recent work [15], statistical trajectories that could be related to the Bohmian trajectories have been observed experimentally. They are smoother than those shown here in Figure 2 and 5. Another difference is that the Bohmian trajectories do not cross the symmetry axis of the system. Those passing on the left (right) of the slits are always deviated to the left (right). This can be seen as a characteristic difference between the Bohmian trajectory that concerns a probability density and the individual trajectory of a single particle.

4. Discussion and conclusion

The results about diffraction that we have summarized demonstrate that there exist in our macroscopic system two waves of the type imagined by de Broglie for quantum mechanics. One of these waves is a pilot wave: the droplet is guided by the local gradient of its phase (Eq. 2) as proposed by de Broglie (Eq.3) for particles. The other wave is a Schrödinger-like probability wave. In this experiment the generation of the probabilistic behaviours can be observed directly, shedding light on the relation between these two waves. Because of its macroscopic scale, our experiment lends itself to performing either intrusive or non-intrusive measurements. The non-intrusive observation of the phenomena associated with an intrusive measurement reveals how it generates a chaotic burst in which both the pilot wave and the droplet trajectory are strongly disturbed [16]. In, e.g., the diffraction experiment, there is a feedback effect. As the wave field becomes more complex (because of the reflected waves) the trajectory bends. In reverse, as the trajectory bends the complexity of the generated wave field increases. However, whatever its complexity, the wave field still has to satisfy the external boundary conditions. There is a statistical sampling of the environment through the propagation of a single walker so that statistical properties are present within the pilot wave. This is why the characteristic interference pattern of the slit shows up in the statistics. More generally, the eigenmodes of the configuration of a detector will contribute to the pilot wave configurations. For this reason a signature of these modes is present in the probability distribution of the particle. In our experiment this effect is entirely related to the path memory effect, a characteristic absent from de Broglie's models. If this idea was transferred to the quantum world where all measurements are intrusive, such an effect could be responsible for both the uncertainty principle and the quantum projection effect. This would require that a wave-mediated path-memory gives a form of spatio-temporal non-locality to quantum particles. To our knowledge this type of hypothesis has not been investigated yet.

References

[1] Faraday M 1831 *Philos. Trans. R. Soc. London* **52** 299-340, On the forms and states of fluids on vibrating elastic surfaces.
[2] Couder Y., Protière S., Fort E., Boudaoud A. 2005 *Nature* **437** 208, Walking and orbiting droplets.
[3] Protière S., Y Couder, and Boudaoud A. 2006 *J. Fluid Mech.* **554** 85-108, A particle-wave association on a fluid interface.
[4] Couder Y. and Fort E. 2006 *Phys. Rev. Lett.* **97** 154101, Single-particle diffraction and interference at a macroscopic scale.
[5] Eddi A., Fort E., Moisy F. and Couder Y. 2009 *Phys. Rev. Lett.* **102** 240401-4, Unpredictable tunneling of a classical wave-particle association.
[6] Fort E., Eddi A., Boudaoud A., Moukhtar J., and Couder Y. 2010 *PNAS* **107** 17515-17520, Path memory induced quantization of classical orbits.

[7] Eddi A., Sultan E., Moukhtar J., Fort E., Rossi M. and Couder Y. 2011 *J. Fluid Mech.* **674** 433-464, Information stored in Faraday waves: the origin of a path memory.

[8] de Broglie L. 1925 *Annales de Physique* **10-III** 22-128, Recherches sur la théorie des quanta. English translation (Kracklauer A.F. 2004), online at Fondation Louis de Broglie, aflb.ensmp.fr/.

[9] de Broglie L. 1927 *Journal de Physique et le radium* **6-5** 225-241, La mécanique ondulatoire et la structure atomique de la matière et du rayonnement.

[10] de Broglie L. 1953 in *Scientific Papers Presented to Max Born on his retirement from the Tait Chair of Natural Philosophy in the University of Edinburgh"* (Oliver and Boyd 1953), English translation (Dileep Karanth 2010) online: http://arxiv.org/abs/1005.4534v1.

[11] de Broglie L. 1959 *Le journal de physique et le radium* **20** 963-979, L'interprétation de la mécanique ondulatoire.

[12] Bohm D. 1952 *Phys. Rev* **85** 166-179 and *Phys. Rev* **85** 180-193, A suggested interpretation of quantum theory in terms of hidden variables. Part 1 and part 2.

[13] At the Solvay conference, de Broglie had admitted that the Schrdinger waves could be the "pilot wave" of the particle. As stated in reference [10] he was later to regret what he considered as a concession.

[14] Philippidis C., Dewdney C. and Hiley B. J. 1979 *Nuovo Cimento B* **52** 15-28, Quantum interference and the quantum potential.

[15] Kocsis S., Braverman B., Ravets S., Stevens M. J., Mirin R., P. Shalm L. K. and Steinberg A.M. 2011 *Science* **332** 1179-1173, Observing the average trajectories of single photons in a two-slit interferometer.

[16] We can note that the chaos we described here has no direct relation to the quantum chaos usually investigated. Most works of this type are devoted to finding a relation between the trajectories of classical particles in, e.g., chaotic stadium-shaped billiards and the behaviour of the Schrödinger waves in the same geometries. These works are thus downstream from Schrödinger probabilities, but do not address their production.